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On Parseval's Equation and Modulation Property for Fractional Gabor Transform

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Abstract: **As generalization of the Gabor transform, the fractional Gabor transform has been used in several areas, including optical analysis and signal processing. In this paper we have proved some important results that are scaling property, Parseval's identity and Modulation property.**

Keywords: **Fourier transform, Fractional Fourier transform, Gabor transform, fractional Gabor transform, testing function space, signal processing.**

I. INTRODUCTION

The Fourier transform is one of the most important mathematical tools used in physical optics, linear system theory, signal processing and so on [1]. The concept of Gabor transform of fractional order studied in [7] using window function. The simple form of fractional Gabor transform of signal $f(x)$ with rotation α is defined as in [4].

$$[G_\alpha f(x)](u) = \mathrm{G}_\alpha(u,t) = \int_{-\infty}^{\infty} f(x) K_\alpha(x,u,t)\,dx \qquad (1.1)$$

Where, $$K_\alpha(x,u,t) = \sqrt{\frac{1-i\cot\alpha}{2\pi}}\, e^{i\frac{(x^2+u^2)\cot\alpha}{2}} e^{\frac{-(x-t)^2\csc\alpha}{2}} e^{-iux\csc\alpha} \qquad (1.2)$$

The above fractional Gabor transform is the generalization of the conventional Gabor transform which is defined as follows in [7]

$$G(u,t) = \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} f(x) e^{\frac{-(x-t)^2}{2}} e^{-iux}\,dx \qquad (1.3)$$

The Gabor transform, STFT and CWT invertibility, and generalized Parseval's like theorem is given in [3]. Parseval's equation for fractional Hankel transform is given in [2]. Modulation and Parseval's theorem for generalized fractional Fourier transform is given in [5].

In this paper section 2 gives scaling property. In section 3 we give Parseval's identity for fractional Gabor transform. In section 4 Modulation property for fractional Gabor transform was proved and section 5 concludes the paper.

Notation and terminology are as given in [6].

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II. SCALING PROPRTY

$$\left[G^{\alpha} f(ax)\right](u)=\sqrt{\frac{1-i\cot\alpha}{a^2-i\cot\alpha}}\,e^{\frac{iu^2}{2}\left[1-\frac{\cos^2\beta}{\cos^2\alpha}\right]\cot\alpha}\left\{\left[G^{\beta} f(x)\cdot e^{\left[\frac{-(x-t)^2+(ax-t)^2\sin\alpha}{2\sin\alpha\cdot\sin\beta}\right]}\right]\left(\frac{\sin\beta}{a\sin\alpha}u\right)\right\}$$

Proof: $\left[G^{\alpha} f(ax)\right](u)$

$$=\int_{-\infty}^{\infty} f(ax)K_{\alpha}(x,u,t)\,dx$$

$$=\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty} f(ax)e^{i\frac{(x^2+u^2)\cot\alpha}{2}}\,e^{-\frac{(x-t)^2\csc\alpha}{2}}\,e^{-iux\csc\alpha}\,dx$$

Put $ax=T$

$$\Rightarrow x=\frac{T}{a} \qquad \Rightarrow dx=\frac{dT}{a}$$

$$=\frac{1}{a}\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty} f(T)e^{\frac{i}{2}\left[T^2+a^2u^2\right]\frac{\cot\alpha}{a^2}}\,e^{-\frac{(T-at)^2\csc\alpha}{2a^2}}\,e^{-iu\frac{T}{a}\csc\alpha}\,dT \qquad (2.1)$$

Let $\frac{\cot\alpha}{a^2}=\cot\beta$ (2.2)

$$\frac{1}{a}=\sqrt{\frac{\cot\beta}{\cot\alpha}}=\sqrt{\frac{1}{\tan\beta\cot\alpha}} \qquad (2.3)$$

$$\frac{1}{a}=\sqrt{\frac{1-i\cot\beta}{\tan\beta\cot\alpha(1-i\cot\beta)}}=\sqrt{\frac{1-i\cot\beta}{\tan\beta\cot\alpha-i\cot\alpha}}=\sqrt{\frac{1-i\cot\beta}{a^2-i\cot\alpha}} \qquad (2.4)$$

$$(2.2)\Rightarrow \frac{\cos\beta}{\sin\beta}=\frac{\cos\alpha}{a^2\sin\alpha}$$

$$\Rightarrow \frac{\cos\beta}{\cos\alpha}=\frac{\sin\beta}{a^2\sin\alpha} \qquad (2.5)$$

(2.1) $\Rightarrow$

$$=\sqrt{\frac{1-i\cot\beta}{a^2-i\cot\alpha}}\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty} f(T)e^{\frac{i}{2}\left[T^2+(au)^2\right]\cot\beta-iTu\left(\frac{\sin\beta\csc\beta}{a\sin\alpha}\right)}\,e^{-\frac{(T-at)^2\csc\alpha}{2a^2}}\,dT$$

….Using (2.2)

$$=\sqrt{\frac{1-i\cot\alpha}{a^2-i\cot\alpha}}\sqrt{\frac{1-i\cot\beta}{2\pi}}\int_{-\infty}^{\infty} f(T)e^{\frac{i}{2}\left[T^2+\left(\frac{\sin\beta}{a\sin\alpha}u\right)^2-\left(\frac{\sin\beta}{a\sin\alpha}u\right)^2+(au)^2\right]\cot\beta-iT\left(\frac{\sin\beta}{a\sin\alpha}u\right)\csc\beta}\,e^{-\frac{(T-t)^2\csc\beta}{2}}\,.\,e^{\frac{(T-t)^2\csc\beta}{2}}\,e^{-\frac{(T-at)^2\csc\alpha}{2a^2}}\,dT$$

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$$=\sqrt{\frac{1-i\cot\alpha}{a^2-i\cot\alpha}}\left[\sqrt{\frac{1-i\cot\beta}{2\pi}}\int_{-\infty}^{\infty} f(T)e^{\frac{i}{2}\left[T^2+\left(\frac{\sin\beta}{a\sin\alpha}u\right)^2\right]\cot\beta}e^{-iT\left(\frac{\sin\beta}{a\sin\alpha}u\right)\csc\beta}e^{-\frac{(T-t)^2\csc\beta}{2}}dT\times\right.$$

$$\left.\left(e^{-\frac{(T-at)^2\cos\beta}{2\cos\alpha\sin\beta}+\frac{(T-t)^2\csc\beta}{2}}\right)\right]e^{\frac{-i}{2}\left(\frac{\sin\beta}{a\sin\alpha}u\right)^2\cot\beta+\frac{i}{2}(au)^2\cot\beta}$$

Using (2.5)

$$=\sqrt{\frac{1-i\cot\alpha}{a^2-i\cot\alpha}}\left\{\left[G^\beta f(x)e^{\left[-(ax-at)^2\frac{\cos\beta}{2\cos\alpha\sin\beta}+(ax-t)^2\frac{\csc\beta}{2}\right]}\right]\left(\frac{\sin\beta}{a\sin\alpha}u\right)\right\}e^{\frac{iu^2}{2}\left[\frac{-\sin\beta\cos\beta}{a^2\sin\alpha\cos\alpha}+1\right]\cot\alpha}$$

Using (2.2)

$$=\sqrt{\frac{1-i\cot\alpha}{a^2-i\cot\alpha}}\left\{\left[G^\beta f(x)e^{\left[-(x-t)^2\frac{\cot\alpha}{2\cos\alpha\sin\beta}+(ax-t)^2\frac{\csc\beta}{2}\right]}\right]\left(\frac{\sin\beta}{a\sin\alpha}u\right)\right\}e^{\frac{iu^2}{2}\left[1-\frac{\cos^2\beta}{\cos^2\alpha}\right]\cot\alpha}$$

Using (2.2) and (2.5)

$$=\sqrt{\frac{1-i\cot\alpha}{a^2-i\cot\alpha}}e^{\frac{iu^2}{2}\left[1-\frac{\cos^2\beta}{\cos^2\alpha}\right]\cot\alpha}\left\{\left[G^\beta f(x)e^{\left[\frac{-(x-t)^2}{2\sin\alpha\sin\beta}+\frac{(ax-t)^2}{2\sin\beta}\right]}\right]\left(\frac{\sin\beta}{a\sin\alpha}u\right)\right\}$$

$$=\sqrt{\frac{1-i\cot\alpha}{a^2-i\cot\alpha}}e^{\frac{iu^2}{2}\left[1-\frac{\cos^2\beta}{\cos^2\alpha}\right]\cot\alpha}\left\{\left[G^\beta f(x)\cdot e^{\left[\frac{-(x-t)^2+(ax-t)^2\sin\alpha}{2\sin\alpha\cdot\sin\beta}\right]}\right]\left(\frac{\sin\beta}{a\sin\alpha}u\right)\right\}.$$

III. PARSEVAL'S IDENTITY

Let $f(x)$ denote a complex valued function of real positive variable x. The fractional Gabor transform of $f(x)$ with parameter α is denoted by $[G^\alpha f(x)](s)$. Parseval's relation for the Gabor transform had given in [3]. We prove the following formulae for fractional Gabor transform which we called Parseval's type formula.

3.1 PARSEVAL'S FORMULA OF FIRST TYPE OF FRACTIONAL GABOR TRANSFORM:

For $g,h\in E'$ we have

$$\int_{-\infty}^{\infty} g(y)h(y)e^{i\frac{y^2}{2}\cot\alpha}e^{-\frac{(y-t)^2}{2}\csc\alpha}K_\alpha(y,s,t)dy=\left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2}\int_{-i\infty}^{i\infty}[G^\alpha g](\omega)[G^\alpha h](s-\omega)e^{i2\omega(s-\omega)\cot\alpha}d\omega$$

Proof:

Consider the left side,

$$\int_{-\infty}^{\infty} g(y)h(y)e^{i\frac{y^2}{2}\cot\alpha}e^{-\frac{(y-t)^2}{2}\csc\alpha}K_\alpha(y,s,t)dy$$

$$=\int_{-\infty}^{\infty}\left\{\int_{-i\infty}^{i\infty}[G^\alpha g](\omega)\overline{K_\alpha(y,\omega,t)}d\omega\right\}h(y)e^{i\frac{y^2}{2}\cot\alpha}e^{-\frac{(y-t)^2}{2}\csc\alpha}K_\alpha(y,s,t)dy$$

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$$= \int_{-\infty}^{\infty}\int_{-i\infty}^{i\infty}\left[G^{\alpha}g\right](\omega)h(y)\overline{K_{\alpha}(y,\omega,t)}K_{\alpha}(y,s,t)e^{i\frac{y^2}{2}\cot\alpha}e^{-\frac{(y-t)^2}{2}\csc\alpha}\,d\omega dy \tag{3.1.1}$$

Now,

$$e^{i\frac{y^2}{2}\cot\alpha}e^{-\frac{(y-t)^2}{2}\csc\alpha}\overline{K_{\alpha}(y,\omega,t)}K_{\alpha}(y,s,t)$$

Using Inversion formula for fractional Gabor transform,

$$= e^{i\frac{y^2}{2}\cot\alpha}e^{-\frac{(y-t)^2}{2}\csc\alpha}\left[\left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2}e^{-i\frac{\left(y^2+\omega^2\right)}{2}\cot\alpha}e^{\frac{(y-t)^2}{2}\csc\alpha}e^{i\omega y\csc\alpha}\right].$$

$$\left[\sqrt{\frac{1-i\cot\alpha}{2\pi}}e^{i\frac{\left(y^2+s^2\right)}{2}\cot\alpha}e^{-\frac{(y-t)^2}{2}\csc\alpha}e^{-isy\csc\alpha}\right]$$

$$= \left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2}\left[\sqrt{\frac{1-i\cot\alpha}{2\pi}}e^{i\frac{\left[y^2+(s-\omega)^2\right]}{2}\cot\alpha}e^{\frac{-(y-t)^2}{2}\csc\alpha}e^{-i(s-\omega)y\csc\alpha}\right]e^{i2\omega(s-\omega)\cot\alpha}$$

$$= \left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2}K_{\alpha}(y,s-\omega,t)e^{i2\omega(s-\omega)\cot\alpha}$$

Using this value in (3.1.1)

$$= \left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2}\int_{-i\infty}^{i\infty}\int_{-\infty}^{\infty}\left[G^{\alpha}g\right](\omega)h(y)K_{\alpha}(y,s-\omega,t)e^{i2\omega(s-\omega)\cot\alpha}\,dyd\omega$$

$$= \left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2}\int_{-i\infty}^{i\infty}\left[G^{\alpha}g\right](\omega)\left[G^{\alpha}h\right](s-\omega)e^{i2\omega(s-\omega)\cot\alpha}\,d\omega$$

3.2 PARSEVAL'S FORMULA OF SECOND TYPE OF FRACTIONAL GABOR TRANSFORM:

For $g,h \in E'$ we have

$$\int_{-\infty}^{\infty}g(xy)h(y)e^{i\frac{x^2y^2}{2}\cot\alpha}e^{\frac{-(xy-t)^2}{2}\csc\alpha}K_{\alpha}(y,s,t)dy$$

$$= \left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2}\int_{-i\infty}^{i\infty}\left[G^{\alpha}g\right](\omega)\left[G^{\alpha}h\right](s-\omega)e^{i\omega(s-\omega)\cot\alpha}e^{i\omega y(x-1)\csc\alpha}\,d\omega$$

This formula is the generalization of the first formula.

Proof:

$$\int_{-\infty}^{\infty}g(xy)h(y)e^{i\frac{x^2y^2}{2}\cot\alpha}e^{\frac{-(xy-t)^2}{2}\csc\alpha}K_{\alpha}(y,s,t)dy$$

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$$= \int_{-\infty}^{\infty} \left\{ \int_{-i\infty}^{i\infty} \left[G^{\alpha} g \right](\omega) \overline{K_{\alpha}(xy,\omega,t)} d\omega \right\} h(y) e^{i\frac{x^2y^2}{2}\cot\alpha} e^{\frac{-(xy-t)^2}{2}\csc\alpha} K_{\alpha}(y,s,t) dy$$

$$= \int_{-i\infty}^{i\infty} \int_{-\infty}^{\infty} \left[G^{\alpha} g \right](\omega) h(y) e^{i\frac{x^2y^2}{2}\cot\alpha} e^{\frac{-(xy-t)^2}{2}\csc\alpha} \overline{K_{\alpha}(xy,\omega,t)} K_{\alpha}(y,s,t) dy d\omega \qquad (3.2.1)$$

Now,

$$e^{i\frac{x^2y^2}{2}\cot\alpha} e^{\frac{-(xy-t)^2}{2}\csc\alpha} \overline{K_{\alpha}(xy,\omega,t)} K_{\alpha}(y,s,t)$$

$$= e^{i\frac{x^2y^2}{2}\cot\alpha} e^{\frac{-(xy-t)^2}{2}\csc\alpha} \left[\left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2} e^{-i\frac{(x^2y^2+\omega^2)}{2}\cot\alpha} e^{\frac{(xy-t)^2}{2}\csc\alpha} e^{i\omega xy\csc\alpha} \right].$$

$$\left[\sqrt{\frac{1-i\cot\alpha}{2\pi}} e^{i\frac{(y^2+s^2)}{2}\cot\alpha} e^{-\frac{(y-t)^2}{2}\csc\alpha} e^{-isy\csc\alpha} \right]$$

$$= \left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2} \left[\sqrt{\frac{1-i\cot\alpha}{2\pi}} e^{i\frac{\left[y^2+(s-\omega)^2\right]}{2}\cot\alpha} e^{\frac{-(y-t)^2}{2}\csc\alpha} e^{-i(s-\omega)y\csc\alpha} \right] e^{i\omega(s-\omega)\cot\alpha} e^{i\omega xy\csc\alpha} e^{-i\omega y\csc\alpha}$$

$$= \left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2} K_{\alpha}(y,s-\omega,t) e^{i\omega(s-\omega)\cot\alpha} e^{i\omega y(x-1)\csc\alpha}$$

Using this value in (3.2.1)

$$= \left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2} \int_{-i\infty}^{i\infty} \int_{-\infty}^{\infty} \left[G^{\alpha} g \right](\omega) h(y) K_{\alpha}(y,s-\omega,t) e^{i\omega(s-\omega)\cot\alpha} e^{i\omega y(x-1)\csc\alpha} dy d\omega$$

$$= \left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2} \int_{-i\infty}^{i\infty} \left[G^{\alpha} g \right](\omega) \left[G^{\alpha} h \right](s-\omega) e^{i\omega(s-\omega)\cot\alpha} e^{i\omega y(x-1)\csc\alpha} d\omega$$

Hence prove,

$$\int_{-\infty}^{\infty} g(xy) h(y) e^{i\frac{x^2y^2}{2}\cot\alpha} e^{\frac{-(xy-t)^2}{2}\csc\alpha} K_{\alpha}(y,s,t) dy$$

$$= \left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2} \int_{-i\infty}^{i\infty} \left[G^{\alpha} g \right](\omega) \left[G^{\alpha} h \right](s-\omega) e^{i\omega(s-\omega)\cot\alpha} e^{i\omega y(x-1)\csc\alpha} d\omega \qquad (3.2.2)$$

3.3 SPECIAL CASES OF PARSEVAL'S TYPE FORMULA (3.2.2)

3.3.1 The special case of (3.2.2) for $s = 1$

$$\int_{-\infty}^{\infty} g(xy) h(y) e^{i\frac{x^2y^2}{2}\cot\alpha} e^{\frac{-(xy-t)^2}{2}\csc\alpha} K_{\alpha}(y,1,t) dy$$

$$= \left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2} \int_{-i\infty}^{i\infty} \left[G^{\alpha} g \right](\omega) \left[G^{\alpha} h \right](1-\omega) e^{i\omega(1-\omega)\cot\alpha} e^{i\omega y(x-1)\csc\alpha} d\omega \qquad (3.3)$$

3.3.2 The special case of (3.3) for $x=1$

$$\int_{-\infty}^{\infty} g(y)h(y)e^{i\frac{y^2}{2}\cot\alpha} e^{\frac{-(y-t)^2}{2}\csc\alpha} K_\alpha(y,1,t)dy$$

$$=\left(\frac{\csc\alpha}{2}\right)\left(\frac{1-i\cot\alpha}{2\pi}\right)^{-1/2}\int_{-i\infty}^{i\infty}[G^\alpha g](\omega)[G^\alpha h](1-\omega)e^{i\omega(1-\omega)\cot\alpha}d\omega$$

IV. MODULATION PROPRTY

4.1 If $[G^\alpha f(x)](u)$ denotes fractional Gabor transform of $f(x)$ then

$$[G^\alpha f(x)\cos ax](u)$$

$$=\sqrt{\frac{1-i\cot\alpha}{8\pi}}e^{\frac{-ia^2}{2}\sin\alpha\cos\alpha}\left\{[G^\alpha f(x)\cdot e^{iau\cos\alpha}](u-a\sin\alpha)+[G^\alpha f(x)\cdot e^{-iau\cos\alpha}](u+a\sin\alpha)\right\}$$

Proof:

$$[G^\alpha f(x)\cos ax](u)$$

$$=\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty} f(x)e^{i\frac{(x^2+u^2)\cot\alpha}{2}} e^{\frac{-(x-t)^2\csc\alpha}{2}} e^{-iux\csc\alpha}\left(\frac{e^{iax}+e^{-iax}}{2}\right)dx$$

$$=\sqrt{\frac{1-i\cot\alpha}{8\pi}}\left\{\left[\int_{-\infty}^{\infty} f(x)e^{i\frac{(x^2+u^2)\cot\alpha}{2}} e^{\frac{-(x-t)^2\csc\alpha}{2}} e^{-i(u\csc\alpha-a)x}dx\right]+\left[\int_{-\infty}^{\infty} f(x)e^{i\frac{(x^2+u^2)\cot\alpha}{2}} e^{\frac{-(x-t)^2\csc\alpha}{2}} e^{-i(u\csc\alpha+a)x}dx\right]\right\}$$

$$=\sqrt{\frac{1-i\cot\alpha}{8\pi}}\left\{\left[\int_{-\infty}^{\infty} f(x)e^{\frac{i}{2}[x^2+(u-a\sin\alpha)^2]\cot\alpha} e^{\frac{-(x-t)^2\csc\alpha}{2}} e^{-i(u-a\sin\alpha)x\csc\alpha}e^{iau\cos\alpha}e^{\frac{-i}{2}a^2\sin\alpha\cos\alpha}dx\right]\right.$$

$$\left.+\left[\int_{-\infty}^{\infty} f(x)e^{\frac{i}{2}[x^2+(u+a\sin\alpha)^2]\cot\alpha} e^{\frac{-(x-t)^2\csc\alpha}{2}} e^{-i(u+a\sin\alpha)x\csc\alpha}e^{-iau\cos\alpha}e^{\frac{-i}{2}a^2\sin\alpha\cos\alpha}dx\right]\right\}$$

$$=\sqrt{\frac{1-i\cot\alpha}{8\pi}}e^{\frac{-ia^2}{2}\sin\alpha\cos\alpha}\left\{[G^\alpha f(x)\cdot e^{iau\cos\alpha}](u-a\sin\alpha)+[G^\alpha f(x)\cdot e^{-iau\cos\alpha}](u+a\sin\alpha)\right\}$$

4.2 If $[G^\alpha f(x)](u)$ **denotes fractional Gabor transform of $f(x)$ then**

$$[G^\alpha f(x)\sin ax](u)=\sqrt{\frac{i\cot\alpha-1}{8\pi}}e^{\frac{-ia^2}{4}\sin 2\alpha}\left\{[G^\alpha f(x)\cdot e^{iau\cos\alpha}](u-a\sin\alpha)-[G^\alpha f(x)\cdot e^{-iau\cos\alpha}](u+a\sin\alpha)\right\}$$

Proof:

$$[G^\alpha f(x)\sin ax](u)$$

$$=\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty} f(x)e^{i\frac{(x^2+u^2)\cot\alpha}{2}} e^{\frac{-(x-t)^2\csc\alpha}{2}} e^{-iux\csc\alpha}\left(\frac{e^{iax}-e^{-iax}}{2i}\right)dx$$

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$$=\sqrt{\frac{1-i\cot\alpha}{2\pi(4i)^2}}\left\{\left[\int_{-\infty}^{\infty} f(x)e^{i\frac{(x^2+u^2)\cot\alpha}{2}}e^{\frac{-(x-t)^2\csc\alpha}{2}}e^{-i(u\csc\alpha-a)x}dx\right]-\left[\int_{-\infty}^{\infty} f(x)e^{i\frac{(x^2+u^2)\cot\alpha}{2}}e^{\frac{-(x-t)^2\csc\alpha}{2}}e^{-i(u\csc\alpha+a)x}dx\right]\right\}$$

$$=\sqrt{\frac{i\cot\alpha-1}{8\pi}}\left\{\left[\int_{-\infty}^{\infty} f(x)e^{\frac{i}{2}\left[x^2+(u-a\sin\alpha)^2\right]\cot\alpha}e^{\frac{-(x-t)^2\csc\alpha}{2}}e^{-i(u-a\sin\alpha)x\csc\alpha}e^{iau\cos\alpha}e^{\frac{-i}{2}a^2\sin\alpha\cos\alpha}dx\right]\right.$$

$$\left.-\left[\int_{-\infty}^{\infty} f(x)e^{\frac{i}{2}\left[x^2+(u+a\sin\alpha)^2\right]\cot\alpha}e^{\frac{-(x-t)^2\csc\alpha}{2}}e^{-i(u+a\sin\alpha)x\csc\alpha}e^{-iau\cos\alpha}e^{\frac{-i}{2}a^2\sin\alpha\cos\alpha}dx\right]\right\}$$

$$=\sqrt{\frac{i\cot\alpha-1}{8\pi}}e^{\frac{-ia^2}{4}\sin 2\alpha}\left\{\left[G^\alpha f(x)\cdot e^{iau\cos\alpha}\right](u-a\sin\alpha)-\left[G^\alpha f(x)\cdot e^{-iau\cos\alpha}\right](u+a\sin\alpha)\right\}$$

V. CONCLUSION

We present scaling property of fractional Gabor transform. The Parseval's identity for the generalized fractional Gabor transform is given. Modulation property is given. Further we plane to prove some more interesting properties of this transform.

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